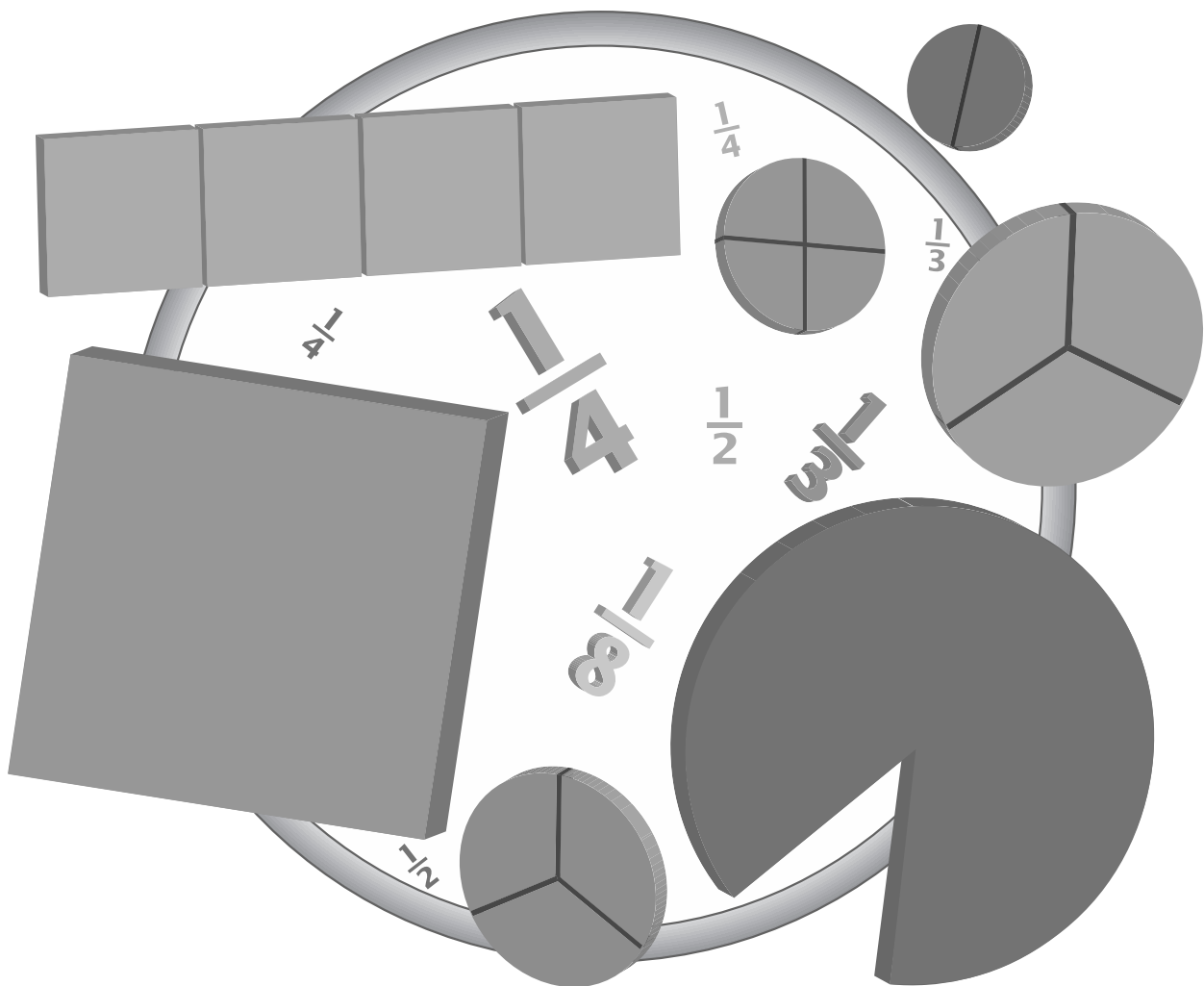


# Fractions

## Made Easy

Understanding Fractions  
with Visual Models



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Standards-Based Instruction, Practice, and Assessment

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# Section 1, Part A: Understanding Fractions

## Common Core State Standard (CCSS) Addressed

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**3.NF.1:** Understand a fraction  $1/b$  as the quantity formed by 1 part when a whole is partitioned into  $b$  equal parts; understand a fraction  $a/b$  as the quantity formed by  $a$  parts of size  $1/b$ .

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### Unpacking the Standard

In third grade, students are expected to develop a basic understanding of fractions. Their understanding is based in the notion of the unit fraction.

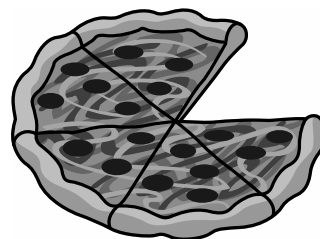
From: Mathematics | Grade 3  
Common Core State Standards for Mathematics – Page 21

(2) Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole . . .

A unit fraction is a single part of a whole that has been divided into many equal parts. Other fractions are built from unit fractions. This standard asks children to recognize unit fractions and larger fractions that are comprised of unit fractions.

For example, in the picture at right,  $5/6$  of a pizza remains uneaten.  $a/b$  in the standard would be represented in this case by the fraction  $5/6$ , where  $a$  is the number of pieces not eaten and  $b$  is the number of pieces in the whole pizza. Similarly,  $1/b$  would be represented by  $1/6$ , the fraction of the pizza that has been eaten.

$5/6$  could also be represented as the total of unit fractions  $1/6 + 1/6 + 1/6 + 1/6 + 1/6$ . Combining unit fractions is similar to combining whole numbers. Just as  $1 + 1 + 1 + 1 + 1$  give a sum of 5,  $1/6 + 1/6 + 1/6 + 1/6 + 1/6$  give a sum of  $5/6$ . A critical concept for students to grasp is that the parts (the unit fractions) are equal to one another.



# Section 1: Understanding Fractions

## Vocabulary

*Fraction* – A fraction is a part of a whole. We refer to fractions in this book as a whole that has been broken into equal parts. A fraction is comprised of equal unit fractions.  $1/2$  has one unit fraction and  $3/4$  has three  $1/4$ -sized unit fractions.

*Unit fraction* – The name given to one piece of a whole broken into equal parts. For example, the unit fraction for a circle broken into three parts is one-third ( $1/3$ ). Unit fractions always have a numerator of 1.

*Whole* – The whole is the entire item that is being broken into parts to form fractions. It represents the value “one.” The whole can be described as an intact item or as a complete set of fractions that form that item. For example, the whole may be an unbroken circle, or it may be a complete circle broken into parts, all of which are being considered (for example,  $4/4$ ,  $6/6$ ,  $8/8$ ).

*Part* – A part is one piece of a whole.

*Equal parts* – For the purposes of the lessons in this section, equal parts are parts that are equal in area. In later sections, parts may be equal according to a different attribute or value, such as equal intervals (lengths) on a number line. Fractions always describe parts that are equal.

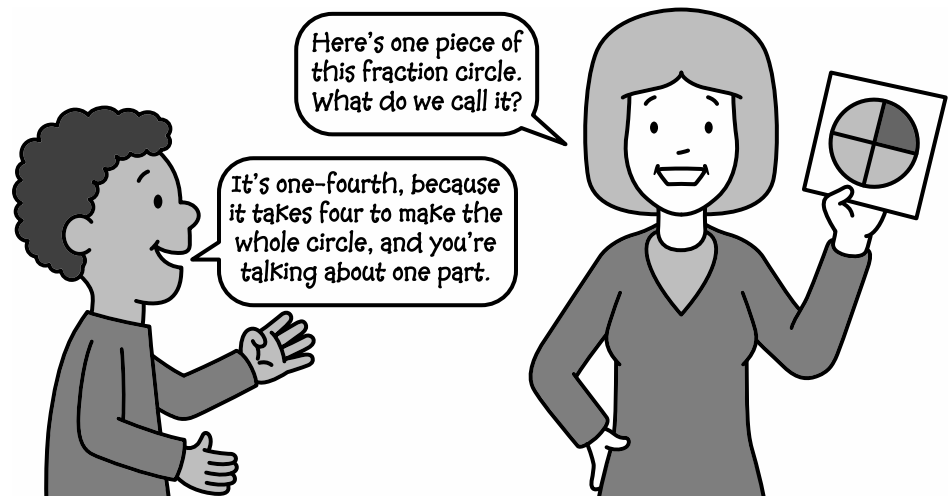
*Part-to-whole relationship* – A part-to-whole relationship requires students to understand that the “whole” has been divided into “parts.” In area models, this is most clearly seen by a shape that has been divided into a given number of same-sized sections (2, 3, or 4 sections). The complete shape is the whole. Each section is a part. In an area model, a number of those parts are typically shaded or colored.

*Numerator* – The numerator is the top numeral in a fraction. It names the number of equal parts being considered. In a unit fraction, the numerator is 1.

*Denominator* – The denominator is the bottom numeral in a fraction. It indicates the number of equal parts in the whole.

*Proper fraction* – Fractions in which the numerator is smaller than the denominator are called proper fractions.  $1/2$ ,  $3/4$ , and  $5/8$  are examples of proper fractions.

*Improper fraction* – Fractions in which the numerator is equal to or larger than the denominator are called improper fractions.  $6/6$ ,  $9/8$ , and  $3/2$  are examples of improper fractions.



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## Notes to the Teacher

This book is based on the premise that learning is most powerful and mathematical standards are most likely to be met when we take our lead from the students. That means we ask the students to inform us about what they know and what they see, rather than informing them about what *we* see. We ask questions to guide their observations and thinking and, in turn, their comments and observations guide our instruction.

The sample questions in this book are offered as suggestions and recommendations only. Learning unfolds differently from class to class and from year to year. You will need to take your lead from your students, their prior understandings, and their particular insights or confusions.

To address the 3.NF.1 standard, Section 1 of this book is divided into two parts.

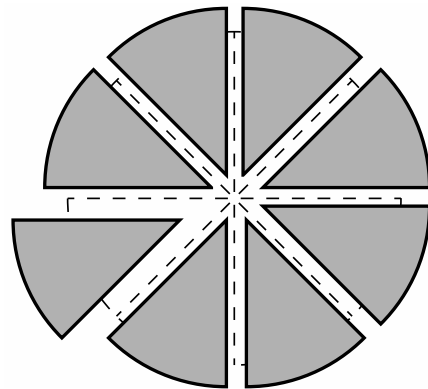
**Part A, How a Fraction Get Its Name** (Lessons 1 and 2) builds essential background in identifying, reading, and writing fractions. These lessons:

- Explore what fractions look like using two-dimensional visual models (rectangle and circle).
- Teach children to read and write fractions as symbols, based on these visual models.
- Lead students to an understanding of the term *fraction* and what the numerals that form the symbolic notation of a fraction (numerator and denominator) represent.
- Culminates in the idea of the unit fraction, which is further developed in Part B.

**Part B, The Unit Fraction** (Lesson 3) extends the learning with a focus on the unit fraction as the building block of all other fraction concepts and operations. This sets the stage for future work with fractions in third grade and beyond.

## Materials for Teaching About Unit Fractions

A visual model that can be manipulated is particularly useful in teaching the idea of the unit fraction. Fraction circles are ideal for teaching this concept. Fraction circles are three-dimensional materials with a circle as the face. The fractional parts are discrete and can be separated. Each discrete part represents a unit fraction. These unit fractions can be compared by placing them next to or on top of one another. The individual parts can be removed from the circle or combined to make a whole.



## Conclusion

We know that as students approach more complex situations with fractions, they may have difficulty seeing the unit fractions involved. Different models and different contexts add complexity, but if the unit fraction concept is solid, connections to more complex situations become simple.



# How a Fraction Gets Its Name

**Note:** You may wish to administer the assessment on pages 26–27 as a pretest to determine whether your students need to learn the skills and concepts in this section.

## Learning Outcomes

- Students will explain the term *fractions* to reflect their understanding of a whole broken into equal parts.
- When shown an area model of a shape divided into equal parts, students will:
  - Identify the number of equal parts as halves, thirds, fourths, sixths, or eighths.
  - Color or shade parts of a whole to show a fraction named by the teacher.
  - Color or shade parts of a whole to show a fraction presented in numeral form (for example,  $\frac{4}{8}$ ).
  - Color or shade parts of a whole shape to show a fraction spelled out (for example, “four-eighths”).
- Students will read fraction names in numeral form ( $\frac{2}{3}$ ) and words (“two-thirds”) and match them to an area model representing fraction.
- Shown an area model divided into equal parts with fractional parts shaded, students will:
  - State the name of the fraction.
  - Write the fraction using numerals in standard form ( $\frac{4}{6}$ ).
  - Write the fraction using words (“four-sixths”).
- Students will identify the numerator and denominator of a fraction and demonstrate or explain what each numeral represents.

## Materials

- *Fractions Made Easy*, pages 6–9

## What to Do

1. Begin the lesson by inviting students to tell you what they already know about fractions. This will help you recognize misconceptions they may have about what fractions are and how they are represented.
2. Direct students’ attention to the rectangle on page 6. (You may hand out copies of the page and/or project it on a whiteboard.)
3. Ask students to identify the shape. (rectangle)
4. Ask students what they notice about the rectangle. Elicit the following information:
  - The rectangle is divided into 4 parts.
  - One of the 4 parts is shaded.
  - The parts are all equal in size. (area)
5. Explain that when something has been divided into equal parts, the parts are called *fractions*.
6. Have students name the fraction:
  - Ask if anyone knows the name of this fraction or would like to guess what its name might be.
  - If no one responds, tell the class that this fraction’s name is “one-fourth.”
  - Ask students why that name might make sense.
7. Have students write the fraction:
  - Invite students to write the fraction using numerals.

- Accept students' answers uncritically and ask them to explain why they think the fraction "one-fourth" might be written the way they chose.
  - If a student offers the correct fraction symbol,  $\frac{1}{4}$ , tell the students that this is the correct way to write the fraction one-fourth.
  - If no student offers the correct fraction symbol for  $\frac{1}{4}$ , show the students the correct way to write the fraction one-fourth. ( $\frac{1}{4}$ )
  - Ask why the notation  $\frac{1}{4}$  makes sense and elicit the following:
    - The 4 below the line represents the four parts of the rectangle.
    - The 1 above the line represents the one part that is shaded.
8. Ask students to consider whether it is important that the parts be equal in area. Elicit the idea (or explain) that the parts must be equal in order to figure out the fraction's name.
9. Review how a fraction is written. Ask students to tell you:
- Where to place the numeral that shows the number of equal parts in the whole rectangle.
  - Where to place the numeral that shows the number of equal parts that are shaded.
10. Move to page 7 to review the ideas developed so far. Guide the students in answering the printed questions. Make sure each student completes the page accurately.
11. As a group, repeat the lesson steps using the fraction  $\frac{3}{4}$  (pages 8–9).
12. Then lead the class in completing Guided Practice page 10 to consolidate their learning.
13. Move on to Guided Practice pages 15–16, which extend students' understanding of how fractions get their names to a two-dimensional circle model. One at a time, discuss with your students the fractions shown on these pages. Use the questions on the page and the sample questions listed below as you lead your students to describe what they see and how they would name and write the fractions shown.
- Note:** You may wish to avoid working with "halves" in these early lessons due to confusion about the name.

#### Sample Questions to Ask:

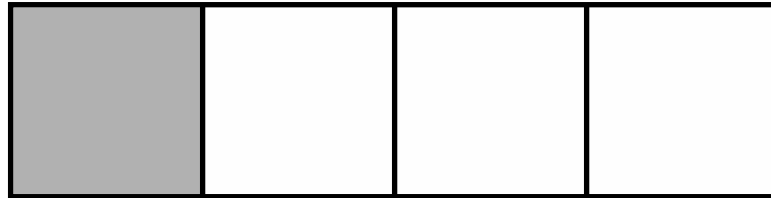
- How many parts in this rectangle?
- Are the parts equal in area? Do you think that matters?
- How many are shaded?
- If we needed to write this fraction, what numeral might we put below the line? Why did you pick that number?
- What numeral might we write above the line? What makes you think that is the correct number?
- Does anyone want to suggest the name for this fraction?
- Why do you think that is its name?
- What do you see? What do you notice?
- How did you get that?
- What makes you say that? What makes you think that?



# How a Fraction Gets Its Name

NAME: \_\_\_\_\_ DATE: \_\_\_\_\_

Use this rectangle to answer the questions below.



- How many parts are in the whole rectangle? \_\_\_\_\_
- Do you think the parts are equal?      Yes      No
- How many parts are shaded?      1      2      3      4
- How many parts are not shaded?      1      2      3      4
- Use the evidence. What is this fraction's name?  
     one-fifth      one-fourth      three-fourths      four-fourths
- Explain your choice: \_\_\_\_\_  
     \_\_\_\_\_
- The name of this fraction is "one-\_\_\_\_\_."
- Write the name of this fraction as a numeral:  
     \_\_\_\_\_



# How a Fraction Gets Its Name

NAME: \_\_\_\_\_ DATE: \_\_\_\_\_

**1. Look at this fraction:**

How many equal parts in all? \_\_\_\_\_

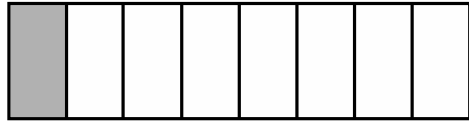
How many parts are shaded? \_\_\_\_\_

Write this fraction's name as a numeral:

$$\frac{\square}{\square}$$

Write this fraction's name in words:

\_\_\_\_\_

**2. Look at this fraction:**

How many equal parts in all? \_\_\_\_\_

How many parts are shaded? \_\_\_\_\_

Write this fraction's name as a numeral:

$$\frac{\square}{\square}$$

Write this fraction's name in words:

\_\_\_\_\_

**3. Look at this fraction:**

How many equal parts in all? \_\_\_\_\_

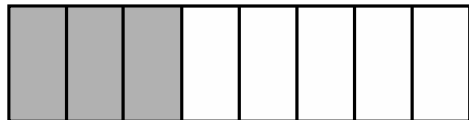
How many parts are shaded? \_\_\_\_\_

Write this fraction's name as a numeral:

$$\frac{\square}{\square}$$

Write this fraction's name in words:

\_\_\_\_\_

**4. Look at this fraction:**

How many equal parts in all? \_\_\_\_\_

How many parts are shaded? \_\_\_\_\_

Write this fraction's name as a numeral:

$$\frac{\square}{\square}$$

Write this fraction's name in words:

\_\_\_\_\_