

Understanding Numbers

The National Council of Teachers of Mathematics states that in grades 3-5 all students should:

- understand the place-value structure of the base-ten number system and be able to represent and compare whole numbers and decimals
- recognize equivalent representations for the same number and generate them by decomposing and composing numbers

Principles and Standards for School Mathematics p. 148
National Council of Teachers of Mathematics

Students who are proficient in mathematics and have the foundational understandings necessary for future success have a deep understanding of place value. They know more than how to tell which digit is in the hundreds place; they know more than how to use an algorithm to get an answer to a computation problem. They understand the structure of the numbers and what happens to the numbers when they add, subtract, multiply, or divide. They understand, for example, that 2,345 is composed of 2 groups of a thousand, 3 groups of one hundred, 4 groups of ten, and 5 ones. They can easily reorganize ones into tens (e.g. 42 ones is 4 tens and 2 ones), tens into hundreds (e.g. 17 tens is 1 hundred and 7 tens), and hundreds into thousands (e.g. 15 hundreds is 1 thousand and 5 hundreds). They use what they know about whole numbers to help them understand decimals. They see that 42 hundredths is worth 4 tenths and 2 hundredths. They know that 7 tens and 5 tens can be combined to form 1 hundred with 2 tens left over and use that information to combine 7 tenths and 5 tenths. They know what quantities are represented by numbers so can easily tell that .30 is smaller than .5.

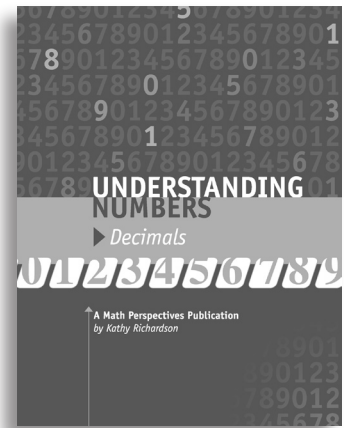
Students who understand the underlying structure of the numbers work with numbers with facility and ease and have the knowledge needed to develop competence with computation.

Developing An Understanding of the Structure of Numbers

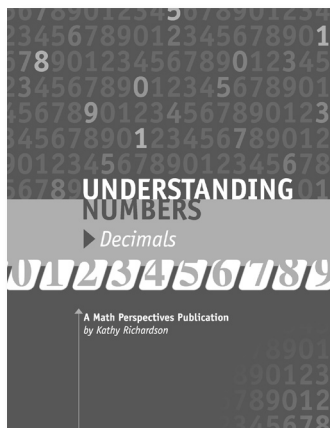
Developing procedures for the addition and subtraction of multi-digit numbers, both whole numbers and decimal numbers, should evolve from the students' understanding of place value.

Principles and Standards for School Mathematics p. 218
National Council of Teachers of Mathematics

Students need many ongoing experiences to develop an understanding of the underlying structure of numbers and the ability to think about numbers flexibly. They need to engage in activities where they group and regroup numbers, break numbers apart, and reorganize them in various ways. They need to find out for themselves that 12 tens, 120 ones, and 1 hundred and 20 ones all describe the same quantities. They need to actually build .12 and 1.2 to perceive the relationship between them.



Understanding Numbers



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The Stations

The Understanding Numbers series of stations are sets of 8 tasks that provide students with the meaningful practice necessary for developing an understanding of the underlying structure of numbers, number relationships and operations. The mathematics presented in the stations is foundational and crucial for developing computational fluency. These foundational concepts also provide the basis for students' understanding of the mathematics they will encounter in the future. Each set of stations presents a variety of activities focused on one major concept. The tasks are designed to meet a range of needs allowing all students to work at their own level. These stations should be experienced over and over again until students have developed proficiency with the tasks. Most students will benefit from working with the appropriate set of stations for several weeks.

Using Models

Students develop an understanding of the structure of numbers by working with models that reveal relationships between numbers. They develop proficiency with the numbers represented by the models when their focus is on identifying the relationships the models reveal rather than on manipulating the models to get answers. They focus on these relationships when they organize and reorganize various models into hundreds, tens, and ones or into tenths and hundredths. So, for example, instead of trading 10 tens for 1 hundred (which can cloud the relationship for some students) they reorganize the tens into 1 hundred. Instead of trading 1 hundred for 10 tens so they can take away 4 tens, they mentally take away or cover up the 4 tens to see that 6 tens are left. They don't count by tens to see what 14 tens are worth, but rather they reorganize the tens into 1 hundred and 4 tens to determine that the blocks are worth 140. When combining 8 and 6, they don't trade 10 ones for a ten, but rather they mentally or physically put 2 of the ones with the 8 to make another 10 and see that 4 are left.

Work with models is effective if it leads students to a level of understanding where they can mentally work with the ideas represented by the models and therefore no longer need the models.

"Research indicates that students' experiences using physical models to represent hundreds, tens, and ones can be effective if the materials help them think about how to combine quantities and, eventually, how these processes connect with written procedures ... The models, however, are not automatically meaningful for students; the meaning must be constructed as they work with the materials."

Adding it Up, p. 198
National Research Council

Differentiating the Tasks to Meet a Range of Needs

... a very important part of the job of a teacher is to guide the child towards tasks where he will be able objectively to do well, but not too easily, not without putting forth some effort, not without difficulties to be mastered, errors to be overcome, creative solutions to be found. This means assessing his skills with sensitivity and accuracy, understanding the levels of his confidence and energy, and responding to his errors in helpful ways."

Margaret Donaldson
Children's Minds p. 120

Students develop a full understanding of important concepts at various times. If each student is to learn all they can, each one must be working at the edge of their own understanding as they work to develop proficiency. The various tasks included in the sets of stations, therefore, are expandable. That is, they can be adapted to meet a range of instructional needs.

Sometimes, having students work with tens and ones rather than with hundreds, tens, and ones, while other students work with numbers in the thousands meets this range of needs. Sometimes the ways that students use models automatically differentiates the task. For example, some students may need to move the blocks or touch the models to solve a problem. Others may need to study the models to aid their thinking but will not need to actually move or touch the model. Others will be able to do the task without the use of a model at all.

Interacting with the Students

Students do not discover or understand mathematical concepts simply by manipulating concrete materials. Mathematics teachers need to intervene frequently as part of the instruction process to help students focus on the underlying mathematical ideas and to help build bridges from the students' work with the manipulatives to their corresponding work with the mathematical ideas or actions.

Jerry Johnson
Teaching and Learning Mathematics, p. 40

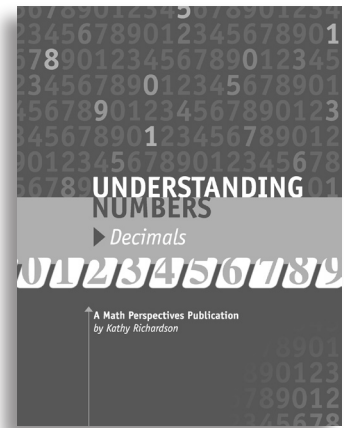
Just as books are the tools that students use to learn to read, the sets of stations are tools for learning mathematics. It is not the task itself that determines what students learn, but rather, what the students are focused on while doing the task that influences the learning. Students will be working with the station tasks independently. However, the teacher has an important role in helping students focus on the important mathematics so that the students learn all they can from their work with the various tasks. The teacher needs to interact with the students while they are at work, observing how they solve problems, questioning them as a way of focusing their thinking, and challenging them. Each station includes suggestions for interacting with students at work.

Introducing the Stations

The whole class can work on these stations at the same time. The stations should be introduced to the whole class over a period of 3 or 4 days. The directions for most of the task are simple and easy for the students to understand. After the tasks have been introduced and the students have had some time to work with the stations, the teacher can begin to adapt the task to more appropriately meet the various needs. Suggestions for adapting the tasks are included in the Notes for Teachers for each task.

Readiness to Work with Stations

Students will be more successful working with the tasks if a culture of self-directed learning and hard work has been established. The students should know how to choose a task, work hard at the task, and move to another task without direction from the teacher. It is worth devoting some time to the development of this way of working.



Understanding Numbers

Decimal Stations

Goals:

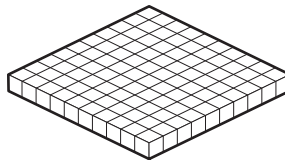
When working with groups of hundredths, tenths, and ones, the students will:

- Count groups of tenths and hundredths
- Recognize the structure of numbers as hundredths, tenths, and ones
- Combine hundredths and tenths by reorganizing the hundredths into all the tenths possible and the tenths into all the ones possible
- Know the number of hundredths that can be made from any group of tenths and the number of ones that can be made from groups of tenths and hundredths
- Combine quantities by reorganizing hundredths into tenths and ones without counting to find the answers
- Compare decimals
- Change one decimal to another by adding on or taking off hundredths and/or tenths

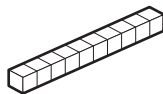
Decimals, fractions, and percents are closely related. It is important that students understand these relationships and the context for using each type of number. This set of stations, however, has a very focused purpose. The tasks in this set of stations give students experiences that help them deepen their understanding of the structure and relative value of decimal numbers. The focus is on tenths and hundredths and forming wholes, with some extensions to thousandths. The students will work with Base Ten Blocks, a model familiar to many of them. For this work, the blocks will represent different values than most of them will have used in the past. To benefit from the stations the student must be able to use the Base Ten Blocks in a flexible way, assigning a different value to them than when they were working with whole numbers.


Students will need to use the term 'one' and 'whole' interchangeably. Referring to the 'one' as a 'whole' helps them focus on the relationships of tenths and hundredths to 1 whole.

10 x 10 Base Ten Block (often referred to as a flat)
is one (the whole).



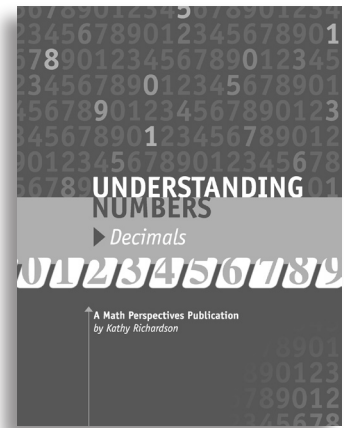
The stick is a tenth.



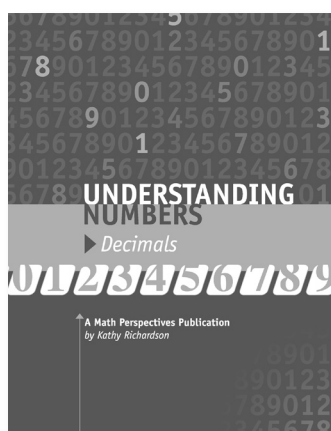
The small cube is a hundredth. 

Marilynn Magnani, teacher of 4th, 5th, and 6th graders shares how she introduces her students to using the Base Ten Blocks to represent decimal numbers.

"I bring out the Base Ten Blocks and ask the students to quietly think about the relationship between the pieces. Then I ask students to share their ideas. I tell them that we are going to use the blocks in a different way than they may have before where the small cube had the value of one, the stick ten, and the flat 100. I ask, 'What if the flat had a value of one? What value would the stick have and what value would the small cube have?' I tell them to think quietly for a minute and then turn to a neighbor and share their thoughts. When the math conversations die down, students share their findings with the rest of the class. Statements are made and vali-



Decimal Stations Introduction



Decimal Stations Introduction

dated using the blocks. Students begin to match up previous concepts of fractional parts and relationships with fractions. I show how to write decimal fractions and we talk about how they fit into the base ten recording system. The key concept that students must build is the relationship of the parts to the whole. The whole determines the value of each part. If the stick is one whole, then the small cubes become tenths because it takes ten of them to make the whole.”

Several of the tasks in this set of stations require the students to determine the total value of a group of decimal numbers by reorganizing various groups of ones, tenths and hundredths. They start with a number of each of these groups. For example: 2 ones (wholes), 16 tenths, and 24 hundredths. They reorganize the 16 tenths into 1 whole and 6 tenths and the 24 hundredths into 2 tenths and 4 hundredths. Next, they combine the ones, tenths and hundredths, forming new groups as needed. Other tasks require the students to add, subtract and compare decimals with the focus continuing to be on using the structure of the numbers to arrive at the answers. Working with the numbers in this way focuses students on the structure and equivalent values of the decimal numbers.

Ongoing Work with Stations

It is through ongoing, repeated experiences with the stations that students develop an understanding of and facility with the structure of decimal numbers. They will move through various stages when working with these tasks. In the beginning some may count or actually move blocks in order to see a relationship. Eventually, they will be able to think about the relationships by looking at the blocks but will not need to move the blocks. Students will benefit from ongoing work with these stations until they can work with decimal numbers with ease and facility without the use of models. This takes time and lots of experiences.

Interacting with the Students

Students work independently with the various station tasks, choosing the particular task they want to do. The teacher’s role during this time is vital. The teacher observes the students while they are at work and interacts with them, asking questions and focusing and challenging their thinking.

When the teacher sees that a student is counting instead of reorganizing into groups of ones (wholes) and tenths, it is more effective to ask a question than to tell the student not to count. The question should engage the student in thinking about the numbers. For example, the teacher might ask, “How many tenths do you have? How many of these tenths do you need to make one whole? How many tenths will be left?” Some students will need to actually rearrange Base Ten Blocks into one whole in order to see how many leftover tenths there are. These same students may still need to count to determine the total. This indicates the underlying structure of the numbers is not yet obvious to the student. The teacher’s questions are not intended to get the student to do a task in one particular way. Some students will not be able to see what the teacher wants them to see. If a student is unable to do what the teacher expects, it is important to allow them to do it in the way that makes sense to them.

A *Guide to Observations Card* that summarizes what to look for and what questions to ask is included. This guide can be kept handy as a reminder for teachers during their conversations with their students.

Adapting the Task

Some students will work with the tasks with ease and will be ready for a challenge. There are two ways that the teacher can adapt the tasks to provide this challenge.

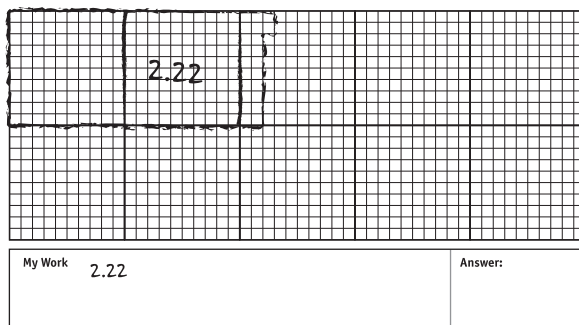
Changing the Whole

If a teacher finds that some students are ready for a challenge, he or she can ask the student to change the value of the blocks. For example, the thousand block can now be worth 1, the flat now worth .1 and the stick now worth .01.

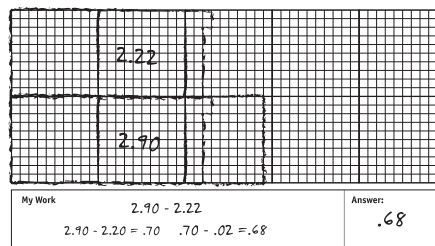
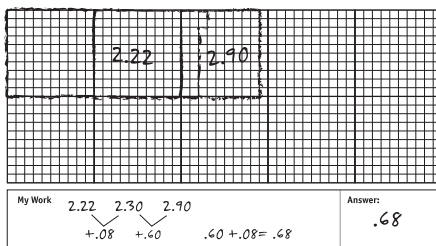
Comparing Numbers

Another way to provide a challenge is to have those students who are ready compare two items to determine the difference between them. The goal is to develop an understanding of what it means to find the difference rather than simply learning to follow the routine procedure of subtracting one number from another. Students can more readily see the relationships between numbers if they use the *Comparing Recording Sheet* and/or the *Thousand Grid Recording Sheet*.

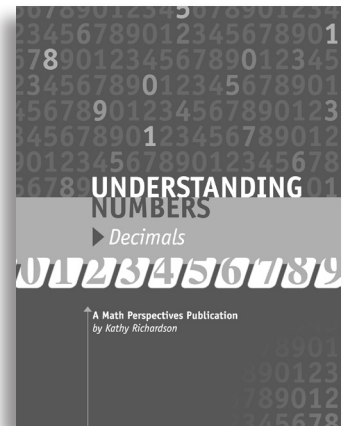
When comparing, students first outline the total value of one of the items they worked with (Buildings, Base Ten Pictures, Paper Shapes, etc).



To compare the other item, they can either outline the value on top of the first outline or use the second row to outline. Students should choose the representation that is easiest for them to understand.



The goal is for the students to eventually learn to compare two items without the aid of the model.



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