Contents

Book-at-a-Glance

This book is a collection of fifteen open-ended math questions for kindergarten, first and second grade children. The questions in this book deal with animals, birthdays, mittens, and other topics that capture the interest of young children.

The problems are open-ended because they have multiple solutions and/or multiple approaches to reach one solution. Appropriate children's literature serves as a springboard for many of the lessons. Classroom examples of typical student responses at each grade level are shown and discussed to provide teachers with possible assessment tools.

The problems are intentionally designed to cover a three-grade span. It is often quite valuable to repeat questions (or present them with minor modifications) throughout the school year and at various grade levels. As students revisit a problem, teachers will gain insights into student growth over time. Each lesson contains the following:

Overview

This is a short synopsis of the lesson that includes a summary of the activity and an approximate time frame.

Materials

Materials necessary for the lesson are listed. These include teacher supplies as well as student supplies. Some items may be marked optional.

Description

This is the "how-to" part of the lesson. Step-by-step details are provided to help teachers implement the lesson in their classrooms. Modifications for particular grade levels are included, when appropriate.

Student Responses and Assessment

A range of student responses is included with each lesson. The responses represent a culturally, linguistically and socio-economically diverse group of children who have actually participated in the lessons across the grade levels. All lessons contain a continuum of **levels of understanding** to offer the teacher tools to evaluate student responses. This section often features questions that the teacher can use when observing children at work and writing anecdotal comments. Suggestions for similar open-ended problems or extensions are also included.

References

Literature selections or related articles are listed in the reference section at the end of each lesson.

> "If we allowed children to show us what they can do rather than accepting what they usually do, we would be in for some grand surprises. As adults, our feeble expectations of children's capabilities puts brakes on their potential."
>
> Mem Fox, *Radical Reflections*

Posing Open-Ended Questions in the Primary Math Classroom

Who has the "answers" in primary grade math classrooms? Children learn at an early age to look for answers from the adults in their lives, rather than from within themselves. In many primary classrooms, young children spend the majority of their time learning that there is one right answer in math; the teacher knows it; and their goal is to find it. Children are rarely asked to describe the strategies that led them to their solution, nor are they asked to examine other possible "right" answers. In fact, when a teacher asks a young child how she has arrived at an answer, the child's first instinct may be to get an eraser because she assumes that she is wrong.

The assumption that grown-ups have all the answers creates a hardship for both children and teachers. Children become one-dimensional in their thinking and are denied the opportunity to become mathematically powerful problem-solvers. Teachers develop a limited view of their students' mathematical progress and are deprived of tools to evaluate the effectiveness of their instructional program.

This is why it is essential that we pose open-ended questions in all math classrooms, beginning in the primary grades. In the process of responding to open-ended questions, children:

- learn to look to themselves for answers and to value themselves as problem solvers;
- build respect for diverse solutions while learning multiple ways of answering questions; and
- develop and express mathematical power.

As students respond to open-ended questions, teachers gain valuable information about each child's:

- ability to think mathematically;
- ease with mathematical communication, orally, pictorially and in writing;
- problem-solving strategies; and
- ability to consider multiple viewpoints and approaches in problem-solving

In addition, student responses provide teachers with insights about their program:

- the overall effectiveness of their math curriculum;
- whether specific lesson objectives have been achieved;
- the appropriateness of the task and the instructional strategies used; and
- whether to modify and/or extend the lesson in the future.

How do teachers encourage children to communicate mathematically? Given that students may be unaccustomed to exploring open-ended questions, it is the teacher's responsibility to create a climate where children are encouraged to share their ideas about mathematics. This task requires risk taking on the part of both adults and children. Teachers need to feel confident that children have the ability to solve their own problems, in a supportive, trusting atmosphere. Children must learn to trust themselves; know that mistakes are a part of the learning process; and recognize that persistence is necessary in order to become powerful mathematicians.

The *NCTM Curriculum and Evaluation Standards for School Mathematics* validates this view:

> *"Ideally, children should share their thinking with other children and with teachers, and they should learn several ways for representing problems and strategies for solving them. In addition, they should learn to value the process of solving the problems as much as they value the solutions."*

To foster a climate conducive to open-ended problem solving, the classroom must encourage risk taking and divergent thinking. Open-ended questions will be most successful in classrooms where:

- Children are encouraged to see themselves as problem solvers, independent of the teacher or textbook.
- Questions are NOT answered immediately by the teacher; rather, children share a variety of approaches and solutions.
- Teachers provide ample time for children to grapple with questions and formulate solutions.
- Teachers model a variety of communication tools —drawings, diagrams, graphs, tables, and oral dictation, as well as written work.
- Support is provided for students to develop mathematical language, through word banks, student-created dictionaries and ample time for discussion.
- Children solve mathematical problems in real-life situations throughout the curriculum, in all subject areas.

It is a challenge to help students communicate mathematically. Initially, when we ask children to explain their answers, they may be intimidated. Many young children think there might be something wrong with their answer. Teachers need to listen carefully to children as they answer these tough questions, and encourage them to be persistent and to look to themselves for solutions. Once young children see themselves as problem solvers, they will take delight in drawing and "writing" about their ideas. Older elementary children can often be resistant to writing in math class because they have never done it before. Younger children, however, have no preconceived notions about what mathematics should be, and enjoy expressing their ideas on paper.

What should teachers expect from student responses to open-ended questions? Expect that your students will respond on many different levels to the lessons in this book. Children bring a wealth of prior knowledge and a natural curiosity to the mathematics classroom, but they vary greatly in their ability to utilize and express this information. The teacher's responsibility is to help each child move forward, to grow in his or her ability to understand and express the concepts that connect to each open-ended question.

It is important to remember that it takes time for students to gain fluency in the language of mathematics. Children need to revisit open-ended math problems throughout the year and in successive years, so that they can develop new insights and approaches to a particular situation.

Recognize that real learning is gradual and often happens imperceptibly. Teachers rarely expect children to master an idea or concept when it is first introduced. The same should hold true as children begin to communicate in mathematics. While revisiting lessons and concepts, thoughtful probing and questioning of students' ideas will gently nudge them to higher levels of understanding. Children need time and multiple opportunities to grapple with, and then embrace, a concept as their own.

Finally, teachers should expect to be delighted and surprised with student responses to open-ended questions. After an initial level of comfort is achieved, students will look forward to solving the problems, sharing their ideas, revising their work and eventually, generating new questions for the class to explore. Teachers will likely discover that there is much joy and excitement in our role as facilitators, as we guide our students to develop true mathematical power.

References

Fox, Mem. *Radical Reflections*. New York: Houghton Mifflin Harcourt, 2001.

National Council of Teachers of Mathematics. *Curriculum and Evaluation Standards for School Mathematics.* Reston, Virginia: NCTM, 1989.

Creating an Open-Ended Math Environment

This section gives you an overview of what you might see in a primary classroom where students are engaged with open-ended problems. It discusses the physical set-up, necessary materials, and the role of children and teachers.

What is the physical set-up of the classroom? First, there should be a place in your classroom for the children to share with the class the discoveries they have made. Preferably, the whole class can sit in this area close enough to hear and see their classmates' work. A carpeted area is nice, but not required. If available, an overhead projector and screen can be used to highlight students' work so that all children can see it. If the children are sitting close together, however, the actual children's work can be viewed by the class without an overhead projector. Individual chalkboards, chalk, and erasers will enhance whole class discussion, when appropriate.

What materials are necessary? Children should have easy access to a range of mathematics manipulatives so that they can explore and represent solutions concretely. These materials should be in labeled containers on low shelves that are also labeled for easy clean-up and return.

The following is a list of the most common classroom math materials necessary to solve these problems:

- unifix cubes or other linking cubes
- pattern blocks or other similar geometric materials
- counting materials such as wooden cubes, tiles, and so on
- junk or treasure boxes
- geoboards

Other materials for specific lessons are listed at the beginning of each lesson. There are also classroom supplies that should be easily accessible to all the children and the teacher:

Children's Supplies

- unlined 8½" by 11" paper
- lined writing paper
- pencils
- crayons
- scissors
- glue or gluesticks
- staplers
- individual chalkboards, chalk and erasers*

Teacher's Supplies

- chart paper
- marking pens
- post-it notes
- overhead transparencies *
- overhead projector *
- screen *

* These supplies are optional.

What should the teacher see as children solve open-ended questions? The classroom should be alive with children thoroughly engaged in their work. Expect noise and clutter as the children delve into these problems. Some children might work alone, while others work with one or more partners. Primary grade children often work best in groups of two, but if the children choose to work in larger groups, allow them to do so. If larger groups are formed, make sure that the children work productively together. Most often, the children are grouped randomly by the teacher. This precludes hurt feelings for the child that seems to always be the last one chosen.

The children choose their work area. Some children prefer to work at tables or desks, while others enjoy working on the floor. The children who work on the floor often choose the individual chalkboards as a backing for their paper as they write.

When the children are thoroughly engrossed in their work, it may be necessary to extend the time. For children to be successful problem solvers, you may need to be flexible with the day's schedule. When most of the class has finished, begin to discuss the task.

What is the role of the classroom teacher? The role of the teacher is two-fold. First, you need to be sure that the problem is thoroughly understood by the children. This does not mean that you teach specific problem-solving strategies; rather, that you check for understanding. You can ask students to re-state the problem in their own words, brainstorm approaches to solve it, and raise questions to clarify the problem. To assure that the problem is accessible to all students, you may wish to present it in a variety of learning modalities—visually, orally and kinesthetically (through dramatic play, modeling).

After the problem is understood, the teacher becomes a facilitator. As the children work, you can circulate around the room to observe children as they solve the problem. You will gain information about children's diverse approaches to problem-solving as well as their ability to work individually or in groups. When children ask questions, you might answer them with another question or redirect the question to the partners or the entire class.

The teacher is also a facilitator during whole class discussions which can occur anytime during the lesson. It is crucial to provide time for discussion after children have completed their work, to allow them to share their strategies and thoughts about the problem. The children explore and talk about mathematical ideas during this time, while the teacher uses questioning techniques to probe and prod the children's thinking to higher levels of understanding.

Overview of The Lessons

Fifteen open-ended problems and additional extensions are described in this book. Each problem addresses the needs of kindergarten, first, and second grade children, and has examples of student work from each grade level.

The following is a list of the fifteen problems and their relationship to the various concepts/strands of mathematics:

Lesson Title	Overview	Concepts & Strands	Literature Connection
1 **Eggs & Chicks**	Children compare the total number of Mrs. Sato's eggs to the total number of baby chicks that hatch.	Number Logic & Language	*Mrs. Sato's Hens* by Laura Min
2 **How Many Snails?**	Children count and sort objects and identify their attributes.	Number Logic & Language Sorting & Classifying	*How Many Snails?* by Paul Giganti, Jr.
3 **How Many Wheels?**	Children use their knowledge of wheels on bicycles and cars to find out how many wheels take children home from a party.	Logic & Language Number	*10 For Dinner* by Jo Ellen Bogart
4 **What Fits Inside the Mitten?**	Children arrange objects by size and determine how many objects will fit in a mitten.	Measurement Sequencing	*The Mitten* by Jan Brett
5 **Crazy Mixed-Up Animals**	Children explore how many animals they can make by mixing up the heads and bodies of animals.	Discrete Math (combinations) Logic & Language Patterns	*All Mixed Up* by Kees Moerbeek
6 **Safari Toss/ Color Toss**	Children collect data from rolling a die and then interpret the results.	Statistics & Probability Discrete Math Number Data Representation	none
7 **Birthday Candles**	Children explore whether there are enough candles for upcoming family birthdays.	Number Logic & Language	none

Lesson Title	Overview	Concepts & Strands	Literature Connection
8 **Sylvester's Pebbles**	Children sort and count rocks in three or more ways.	Sorting Data Representation	*Sylvester & the Magic Pebble* by William Steig -or- *Everybody Needs A Rock* by Byrd Baylor
9 **The Duckling Problem**	Children create stories and/or number sentences about what would happen if the Mallard's ducklings wander away.	Create-a-Problem Representing Numbers Subtraction Addition	*Make Way For Ducklings* by Robert McCloskey
10 **Pattern Block Puzzles**	Children use geometric materials to create and solve riddles and puzzles.	Geometry Logic & Language	*Shapes, Shapes, Shapes* by Tana Hoban
11 **Unifix Trains**	Children find possible combinations of three or four different colored cars for a train.	Discrete Math (combinations) Patterns	*Freight Train* by Donald Crews
12 **Measuring Neil's Desk**	Children explore what happens when they measure a desk with objects of various sizes.	Measurement Number Comparing	*The Line Up Book* by Marisabina Russo
13 **Read-a-Graph**	Children interpret data from a graph.	Statistics Logic & Language	none
14 **Geoboard Geometry**	Children create pictures or designs on a geoboard and describe what they've made.	Spatial Sense Geometry Logic & Language	none
15 **Sweet Treats**	Children create their own math investigations using snack-sized bags of candies.	Logic Data Representation Number	*The M&M's Counting Book* by Barbara Barbieri McGrath

Eggs and Chicks

Materials

- Unifix cubes, wooden cubes, or other counting materials
- *Mrs. Sato's Hens* by Laura Min
- Plastic or paper eggs as counters (optional)
- Unlined paper
- Pencils, crayons, and/or colored marking pens
- Overhead projector (optional)
- Screen (optional)
- Overhead transparencies of children's work (optional)
- Post-it notes (optional)

Overview

This lesson can be a good place for you and the children to begin because it focuses on an area of mathematics in which students and teachers are usually most comfortable — Number. If Mrs. Sato and her friend found two eggs on Monday, three eggs on Tuesday, four eggs on Wednesday, five eggs on Thursday, and six eggs on Friday, how many eggs did they find? First and second grade children are asked to find the total number of eggs and decide if all the eggs hatched at the end of the story, while kindergarten children simply find the total number of eggs. Allow at least 40 minutes for the children to hear the story and solve the problem.

Description

The springboard for this problem is the book *Mrs. Sato's Hens* by Laura Min. In this story, a young girl visits Mrs. Sato and helps count her eggs. On Monday they counted two white eggs, on Tuesday they counted three brown eggs, on Wednesday they counted four speckled eggs, on Thursday they counted five small eggs, and on Friday they counted six big eggs. Finally on Saturday they didn't count any eggs and the picture shows Mrs. Sato and her friend surrounded by baby chicks.

Gather the children around you as you read the story to the class twice. Count together the number of chicks that are shown on the last page. There are fifteen. Have the children retell the facts of the story as you write the following numbers and problem on the chalkboard:

2 white eggs
3 brown eggs
4 speckled eggs
5 small eggs
6 big eggs
15 chicks

Did all the chicks hatch? Show and tell how you solved this problem.

Discuss the problem and give the children the opportunity to clarify the question. Explain that they can use any of the materials in the classroom to find out if all the chicks hatched. You may want to have paper or plastic eggs for them to count. Give them

blank paper to record (using pictures and/or words) their answers so they can report back to the class at a later time. Tell them they may work alone or with a partner, but each child needs to prepare his or her own "report."

As the children work, circulate throughout the room observing them. Use this time to encourage those children who stay on task and to refocus those who need it. Designate a place for them to put their finished work and a task for them to do when the work is complete. When all the children have finished, remind them that they will get a chance to share their reports later.

Sometime before the next part of this lesson, look through the children's work for samples you can make into overhead transparencies. Choose the samples of work that are obviously on the right track and/or have a unique way of approaching the problem. Ask the children whose work you chose if they are willing to explain their solution(s) to the rest of the class.

Finally, have the class gather together around the overhead projector and ask the children to share their work. Be sure to emphasize the positive aspects of each child's report and have the child explain his or her thinking. Discuss the samples and provide the opportunity for the children to revise their work based on the information gained from the samples that were shared.

In kindergarten, the lesson would be similar, but without the expectation of writing about the answer. Children may choose to record what they find out by drawing a picture, and the teacher or another adult can take dictation on post-it notes or the back of the picture about the child's thinking. Read the story initially in a large group. Then reread the story and have the children work on the problem in small groups. Counting materials such as plastic eggs, unifix cubes, or paper eggs would be helpful for K's. Expect that the task of counting the eggs to twenty will be a difficult one for many kindergarten children, especially if you choose to do this task during the first half of the school year.

Student Responses and Assessment

In responding to this problem, some children chose to represent the eggs by drawing. Some used unifix cubes or pattern blocks, while others made tally marks. Reports also varied — some children simply drew their answer, others used words and pictures, and a few children only wrote about their solutions.

The samples shown are from the first month of school in a kindergarten class and a combination first and second grade class. The results might be much more sophisticated from either class later in the year, yet both teachers considered the lesson a success. It is a problem that all the children felt comfortable attempting to solve. The discussion after the children completed their responses gave the class additional information, and the children in both classes were allowed to change their responses based on the information.

Small motor development in young children varies tremendously from child to child. Some children may have marvelous ideas about how to solve a problem, but very little skill at putting those ideas down on paper. There are several ways to record children's ideas other than having them always draw or write about the solution. Consider doing some of the following as your children work on this and other open-ended problems:

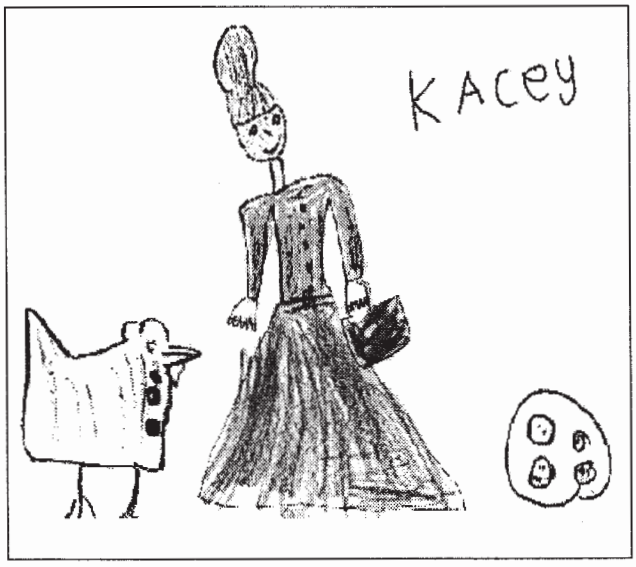
- Take anecdotal records as the children work, either using post-it notes on a clipboard to later attach to the child's work or actually writing on the back of the child's paper.
- Have a camera ready to take pictures of the children at work as they solve the problem.
- Use a tape recorder to have children elaborate on their written report, either as they share with the class or individually.
- Videotape the children as they are working and as they share.

In the classroom, expect the responses to vary. Most kindergartners, and some first graders will find just keeping track of the twenty eggs to be a challenging task. Kindergarten children, if this is their first experience solving a problem of this nature, may focus on other aspects of the story rather than the number of eggs. It might be helpful to consider **levels of understanding** that children have when answering this question. These are (from the most basic to the fullest grasp of understanding):

Levels of Understanding

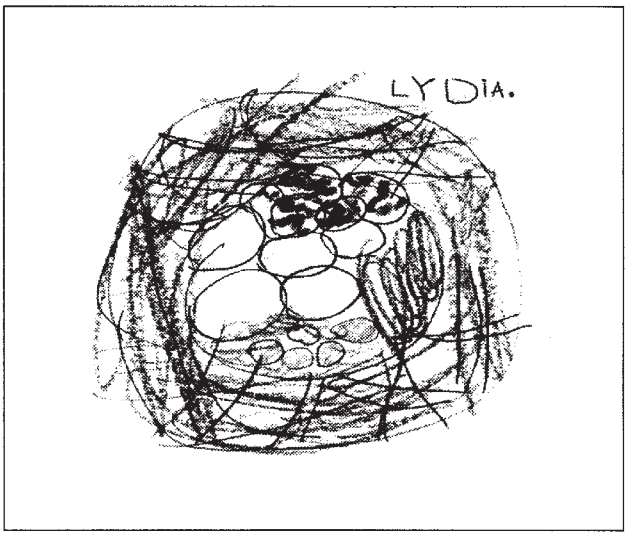
- **The child demonstrates a simple understanding of the story with no reference to mathematics.**
- **The child depicts some of the basic mathematical elements of the problem.**
- **The child depicts most of the mathematical elements of the problem.**
- **The child demonstrates knowledge of more or less in relation to the eggs and chicks.**
- **The child demonstrates knowledge of more or less in relation to the eggs and chicks and uses the number five to express the difference.**

Consider these levels as we look at some student work.

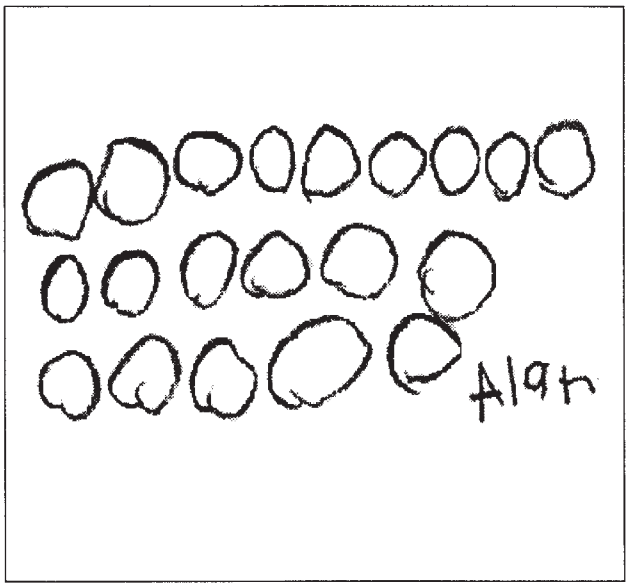


Kacey, a kindergarten child, demonstrates a simple understanding of the story with no reference to the mathematics. In her work, she carefully drew and told about the story without mentioning the number of eggs. She dictated to her teacher: "The chicken was crying because he thought Mrs. Sato was going to take the eggs. She wanted the eggs to hatch." As with many students new to this problem-solving process, Kacey was enthralled with the story and paid little attention to the math she was asked to do. Some students this age, like Kacey, find it hard to hold both the story and the problem in mind at the same time.

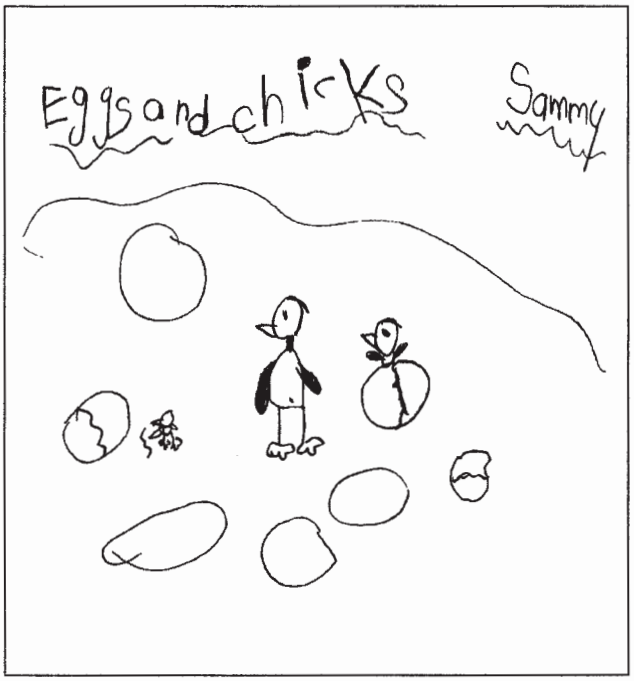
In the same kindergarten class, however, many children were able to tell there were twenty eggs in all, and some even tried to compare this with number of chicks that hatched. Both Lydia and Alan attempted to explain what happened to the eggs that did not hatch.



Lydia depicted most of the basic mathematical elements of the problem, but did not draw all the eggs or state the total number of eggs. She drew only 15 eggs, but said that there were more eggs than chickens: "The four speckled eggs, the five little eggs, the white big eggs. There were more eggs than chickens. Maybe they were eating eggs."

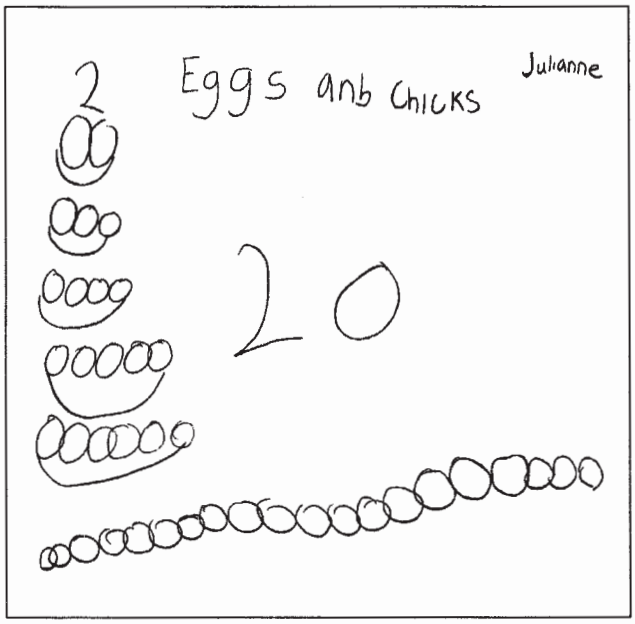


Alan demonstrated knowledge of "less" in relation to the eggs and chicks by implication, but did not give the exact difference. He showed the total number of eggs in his drawing and knew the number of chicks. Alan dictated to his teacher: "There was twenty eggs and only fifteen chicks. I think the other eggs rolled away."

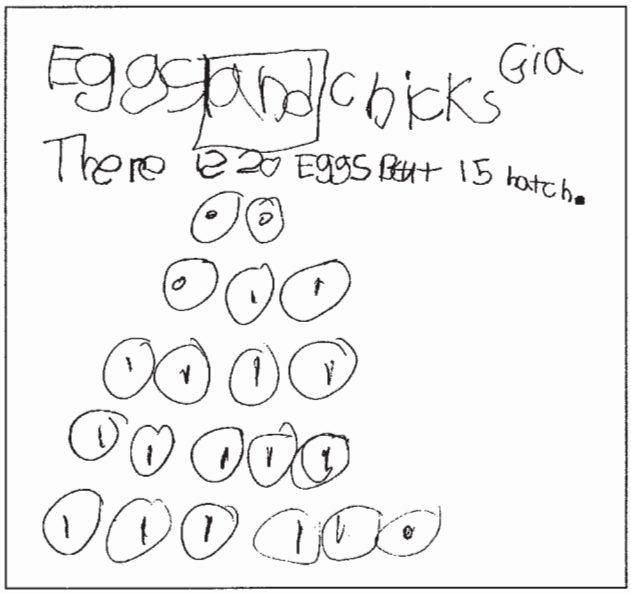


In first and second grade, also expect a variety of ways of looking at the problem.

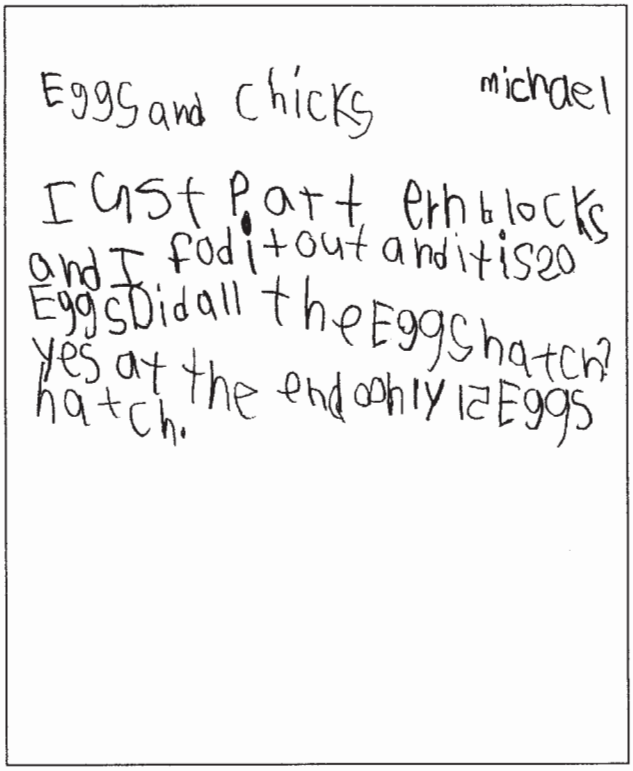
Sammy, a first grader, demonstrated a simple understanding of the story with no reference to mathematics. His level of understanding was basic, even though his teacher gently prodded him about the number of eggs while he was working.



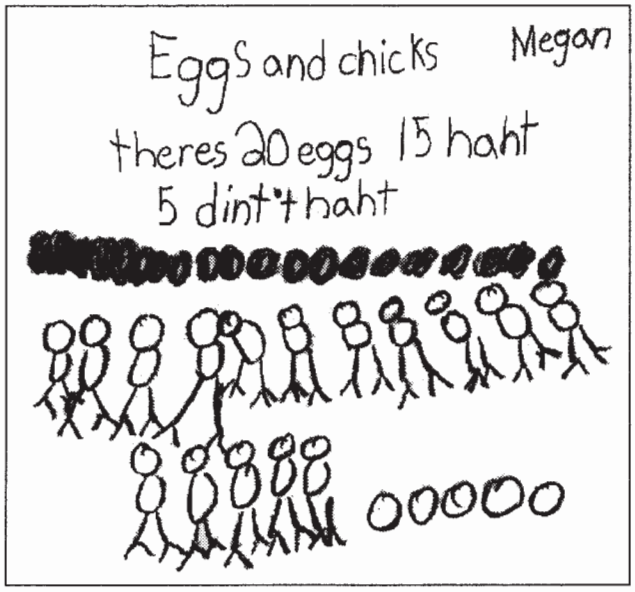
Julianne, another first grader, depicted some of the elements of the problem. She was able to tell the number of eggs, but did not elaborate on the number of the chicks when she shared with the class.



Gia, a first grader, depicted most of the mathematical elements of the problem. She drew and counted the correct number of eggs, and even stated that only 15 hatched.

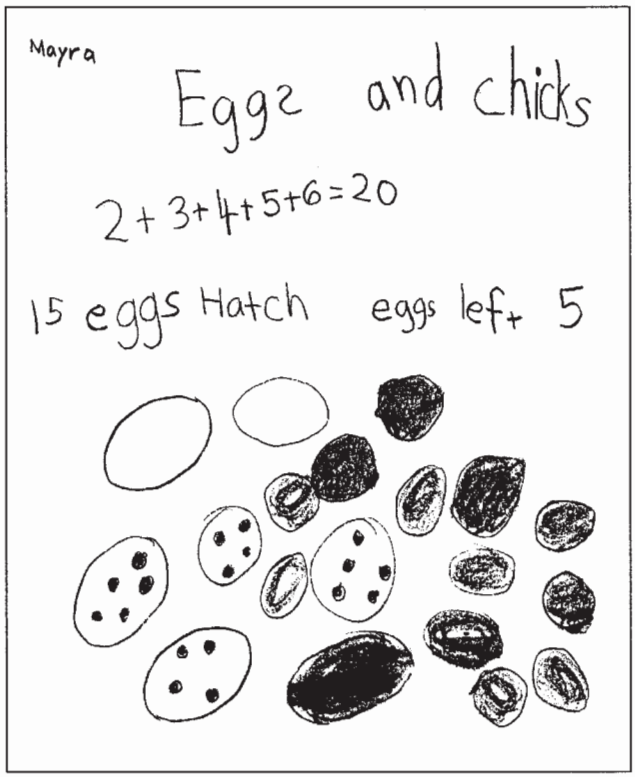


Michael, while sharing his work with the class, demonstrated his knowledge about the difference between the chicks and eggs when he said he felt all the eggs hatched later, even though only 15 hatched at the end of the book. His sharing enhanced our understanding of Michael's view of the problem. In his writing and his sharing, no mention was made of the difference between the number of eggs and the number of chicks.



Megan illustrated the chicks and the five unhatched eggs. Note that her picture contains an extra chick, but her writing says, "15 haht."

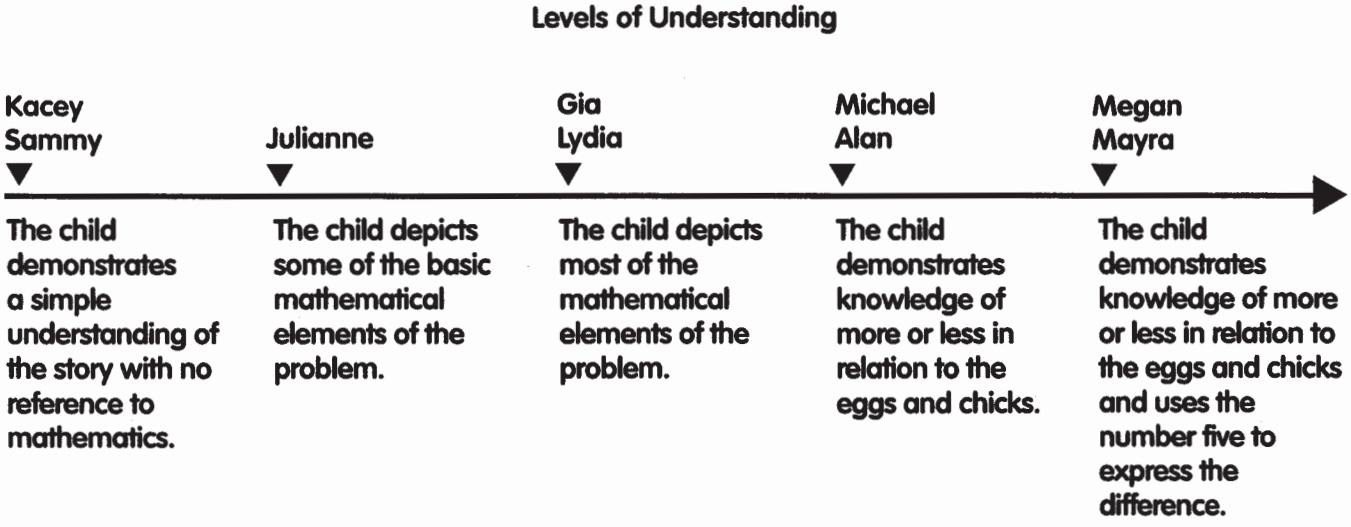
Both Mayra and Megan demonstrated a knowledge of more or less in relation to eggs and chicks and used the number five to express the difference.



Mayra included an equation and drew each kind of egg.

Because both of these second grade children were able to find the difference between the two amounts, they demonstrated a high level of understanding of the mathematics in this problem.

If we were to plot the children on a horizontal continuum according to their **levels of understanding** for this problem, it might look like this:



If this is a first experience at solving problems like this, keep in mind that children who are not comfortable initially need more experiences of this type before we can assess them accurately. Trust that children will become better problem-solvers as their experiences with these open-ended problems increase. Your goal should be to move each child to higher levels of mathematical understanding throughout the year. Young children need many opportunities to grapple with open-ended problems to make the mathematics become real for them.

Extensions

A similar problem to try uses the book *Too Many Eggs* by Christine Butler. In this book, Mrs. Bear puts too many eggs in a birthday cake that calls for only six eggs. The problem would be:

> **Mrs. Bear first used five eggs, then added six eggs and finally added nine eggs to her cake batter. The recipe called for six eggs. How many extra eggs did she add?**

Both these problems, but especially the one about *Mrs. Sato's Hens*, would fit nicely in an integrated math/science unit on oviparous animals. Hatching eggs in the classroom would be the perfect opportunity to see if, in real-life, all the eggs do hatch.

References

Butler, M. Christine. *Too Many Eggs*. Boston: David R. Godine, 1988.

Min, Laura. *Mrs. Sato's Hens*. St. Paul, MN: Globe Publishing, 2000.