Book Three

Developing Number Concepts

Place Value, Multiplication, and Division

Kathy Richardson

Math Perspectives

Table of Contents

CHAPTER ONE (continued)

Table of Contents

Developing Number Concepts—the Series

Developing Number Concepts is a series of books designed to help young children develop important foundational mathematics concepts.

Each of the three books in the series includes cohesive and organized sets of experiences focused on particular mathematical ideas. Every concept is developed both through teacher-directed and independent activities. Because children learn at different rates the activities are "expandable" and, therefore, meet a range of needs. Questions that guide teachers' observations of children as they work and learn help in the assessment of children's ongoing progress.

Book One

Chapter 1: ***Beginning Number Concepts***
Chapter 2: ***Pattern***
Chapter 3: ***The Concepts of More and Less***

Book Two

Chapter 1: ***Interpreting and Symbolizing Addition and Subtraction***
Chapter 2: ***Internalizing Number Combinations to 10***
Chapter 3: ***Developing Strategies for Adding and Subtracting***

Book Three

Chapter 1: ***Place Value***
Chapter 2: ***Beginning Multiplication***
Chapter 3: ***Beginning Division***

The Planning Guide for Developing Number Concepts accompanies the series. It is for the use of teachers of kindergarten through grade three and teachers of multi-grade classes. It includes comprehensive year-long teaching plans along with classroom management ideas.

How the Books Are Organized

Each chapter of *Books One, Two,* and *Three* includes the following.

■ **What You Need to Know About...**

This section provides the teacher with background information on the featured math concept and a summary of ways in which to teach the concept.

■ **Chapter Overview**

A brief overview of the chapter follows. It offers pertinent information on how the math concept should be taught to children at each grade level, kindergarten through grade three, and to children with special needs.

■ **Goals for Children's Learning**

This section lists the mathematics concepts, ideas, and skills that the children will learn as they work with the activities.

■ **Analyzing and Assessing Children's Needs**

Questions to guide teachers' observations and a discussion of how the activities can be used to meet a range of needs are included. The questions are geared to help teachers determine if the tasks that children are working with are appropriate and are meeting their needs.

■ **Classroom Scenes**

Realistic classroom scenes that deal with the major math concepts covered in the chapter help bring the activities to life as they model ways in which the teacher can work.

■ **About the Activities**

Included here is a brief discussion about the purpose of the activities along with information about materials preparation.

■ **Teacher-Directed Activities and Independent Activities**

A great variety of both teacher-directed and independent and/or partner activities are included for each math concept. This gives teachers many different ways to meet children's needs while it gives children many different ways to learn about a particular concept.

■ **Blackline Masters**

Blackline masters, used both for materials preparation and as children's worksheets, appear at the end of each book.

What You Need to Know About Place Value

Place-value concepts are not easy for young children to grasp. Many children, exposed to these ideas too soon, too quickly, and too abstractly, remain uncertain and confused about them throughout elementary school. Year after year, children ask their teachers questions like, "Do we have to borrow?" and "Is this when we carry?" Children diligently memorize steps and rules for getting answers, often forgetting or misapplying them. The teacher's job in the primary grades is to build a foundation so that children will be able to make sense of place value and determine for themselves whether or not they are getting reasonable answers when solving problems. The following are several ideas that children must deal with as they come to terms with place-value concepts.

When we count large numbers, we keep forming new groups based on tens.

The most basic concept confronting children is that our number system is based on forming groups of ten. That is, when we have ten units, we group them into one group of ten. When we have ten groups of ten, we group them into one group of one hundred. When we have ten groups of one hundred, we group them into one group of one thousand, and so on.

We count groups as though they were single objects.

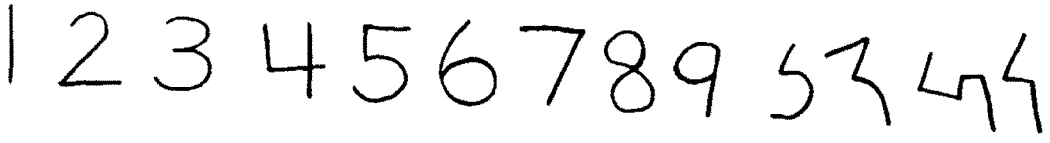
For children who have been counting single objects, counting groups requires a shift in thinking. Children's first counting experiences involve an understanding of one-to-one correspondence. They learn that each count can be matched to one object. But when dealing with numbers beyond ten, they are required to count groups as though they were individual objects.

We can look at one amount as being made up of many different groups.

Children need to understand that ten ones is the same as one group of ten, and that 100 ones is the same as one group of 100 or ten groups of ten. Posing the question, "How many tens are there in 34?" assumes that the child can conceive of ten objects as one entity. Posing the question, "How many hundreds are there in 346?" assumes that the child can conceive of 100 objects as one entity (all the while remembering that each hundred is also ten groups of ten).

A numeral (digit) can stand for different amounts, depending on where it is written in the number.

Another key idea the children must learn is that a particular numeral can stand for many different amounts, depending on its position, or *place,* in a number. Imagine for a moment what it would be like if every number we could think of had to be written with a unique symbol—such a thought is overwhelming.



Teaching and Learning About Place Value

The activities in this chapter help children develop an understanding of place-value concepts including forming and counting groups, recognizing patterns in the number system, organizing groups into tens and ones, and adding and subtracting two-digit numbers. The chapter is organized into three sections. Each section deals with important concepts that children must understand if they are to make sense of our number system and learn to add and subtract large numbers in a meaningful way.

In each section, you will introduce the concepts to the children through teacher-directed whole-class activities. You will provide follow-up in the form of independent activities that give children the opportunity to consider and apply these ideas for themselves.

Section A: Understanding Regrouping—The Process and the Patterns The whole class is introduced to the underlying concepts of place value through a set of activities called the grouping games, through which children practice forming and counting groups. They record and examine the patterns that emerge from this process. The children then build on this experience as they work with groups of ten, interpreting numbers as "tens and leftovers," and searching for patterns in base-ten numbers.

Section B: Developing a Sense of Quantities to 100 Children develop a sense of quantities to 100 and come to recognize the value of organizing numbers into groups of tens and ones. They learn to take numbers apart in a variety of ways, a process that develops flexibility with numbers and an understanding of conservation of large numbers.

Section C: Addition and Subtraction of Two-Digit Numbers Children learn to interpret addition and subtraction problems with manipulative materials and develop their own strategies for determining sums and differences.

Using the Chapter

Your use of this chapter will vary according to the needs of your children. The "Meeting the Needs of Your Children" charts in the introduction to this book and in the *Planning Guide* that accompanies this series offer detailed information that can help you plan how to use the chapter's activities. The following are general suggestions for using the activities with different groups of children.

Kindergarten The place-value concepts in this chapter are inappropriate for use in kindergarten. While it may be of some value to children to experience organizing objects into groups of tens and ones in situations like counting pumpkin seeds, the children would be generally learning to use the place-value language without really understanding the underlying concepts.

First Grade Although many first-grade children can count to 100 and are intrigued by looking for number patterns, most first graders will not be able to understand place-value concepts. In order to understand place value, they must be able to think of ten in two ways at the same time, both as one ten and as ten ones. (The difficulty that this kind of thinking presents to young children has a parallel in a more common situation: at a certain stage of thinking, young children can't accept the idea that their mother is both their mother and their grandmother's daughter at the same time.) First graders' work with large numbers is, for the most part, focused on counting. Such work prepares them for later work with place-value concepts. If you introduce place-value concepts to your first graders, be aware that many will not understand the concepts or the language they are learning to use.

Second Grade Place-value concepts are the core of the second-grade mathematics program. Second-grade children will explore the ideas of forming and counting groups and discover the patterns that emerge from this process. They will have opportunities to discover the patterns in the place-value system and will see how these patterns occur in their environment. They will engage in many different activities that require them to work with tens and ones and that help them to develop a sense of quantity of numbers to 100. They will also work with addition and subtraction of two-digit numbers and will develop their own strategies for determining the sums and differences.

Third Grade The amount of work that third-grade children need to do with the place-value concepts presented in this chapter will depend in large part on their previous experiences. Most third-grade children will benefit from working with these concepts in the beginning of the year. Extend the activities so that children are working with numbers beyond 100 and are adding and subtracting three-digit numbers.

Children with Special Needs Because place-value concepts are difficult for children to understand, we often ask children simply to memorize procedures rather than giving them sufficient time to develop understanding. Do not rush children through these experiences. They will be better served in the long run if they are allowed to make sense of what they are learning. Provide them with lots of experiences organizing groups into tens and ones. Give the children a great deal of practice counting groups of tens using real things. Give them many opportunities to interpret and build models of two-digit numbers using beans and cups or connecting cubes.

SECTION A

Understanding Regrouping: The Process and the Patterns

The activities in this section help children understand the underlying concepts of place value so that they will be able to work effectively with our base-ten number system. Children form groups in a systematic way, counting and recording the groups and the leftovers and then identifying the number patterns that emerge.

Instead of beginning to work with groups of ten, the children will work forming smaller groups of four, five, and six. This is because working with groups of less than ten enables children to focus more easily on the process of regrouping, since they are forming groups more often than if they were working with groups of ten. Later, when the children work with tens and hundreds, the regrouping process will already be clear to them.

The children also learn to count groups and to write the numbers that stand for those groups in the appropriate places. They search for the patterns that emerge when forming groups in a regular fashion. The repetitions in the patterns that emerge when working with groups of four, five, and six are shorter and thus more easily recognizable than similar patterns in base ten. The experiences identifying these shorter patterns will help the children later when they begin to look for patterns in base ten.

Once the foundation has been laid using smaller groups of numbers, the children apply what they have learned as they work with groups of ten. The patterns in the base-ten number system then become the focus of study as the children discover and work with a variety of patterns in the numbers to 100 and beyond.

Goals for Children's Learning* *(Section A)*

Goals

Given a variety of situations, the children will:

- Form and count groups
- Interpret numbers as groups and leftovers
- Interpret numbers as tens and ones (or hundreds, tens, and ones) using models
- Identify patterns in sequences of numbers
- Find numbers with ease on the 00–99 chart
- Count and write numbers to 100 and beyond

* Adapted from *How Do We Know They're Learning? Assessing Math Concepts*.

Teacher-Directed and Independent Activities

About the Grouping Games

The grouping games teach the children a particular process for forming and counting groups. You will go through this process step by step with the children, using either connecting cubes or beans and cups with the place-value boards, until you are sure that the children understand the procedure and can apply it to whatever size group they are working with.

When you first introduce the grouping games, the children work with groups of four. Later they will work with groups of five and six. As a class, they pick a nonsense word to name each group. Naming a group gives it an identity and helps the children think of it as one entity. While introducing a nonsense word into mathematics may seem unnecessary, referring to a group of objects in this way is actually less confusing to children than referring to it with a number like *four* or *five*.

1–1 Introducing the Plus-One and Minus-One Games* — Whole-Class Activity

Materials: Connecting cubes (sorted by color) or beans and cups ▪ Place-value boards (1 per child). (See Materials Preparation, p. 14.)

The Plus-One Game

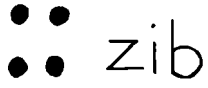
The first task is choosing a word to describe the group you are going to work with. Say, for example:

Today we are going to play a counting game. We will be counting groups of cubes (or beans). In this game, we are going to make groups of four, but we can't say the word "four." We need another name for these groups, so we have to make up a new word that means "four." The word can't mean anything else. Who has an idea?

I have an idea. "Zib."

(This is just an example of a nonsense word. Your students will invent their own word.)

OK, "zib." I will write it on a chart so we won't forget.

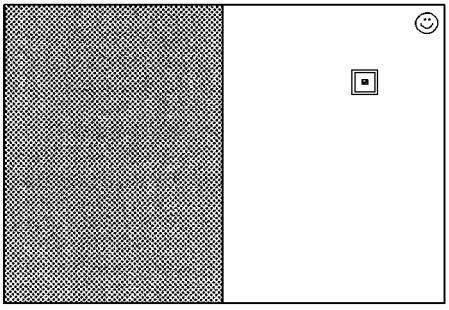


* Based on *Mathematics Their Way,* "Counting Game: Concept Development Stage," p. 276.

To start the Plus-One game, each child should have a place-value board and a supply of connecting cubes.

When we count zibs, we will use our place-value boards to help us organize our counting. The green side of the board will be where we put the zibs, and the white side will be where we put loose cubes when we don't have enough for a zib.

When I say "Plus one," I want you to put one cube on the white side of your board. Okay, "Plus one."



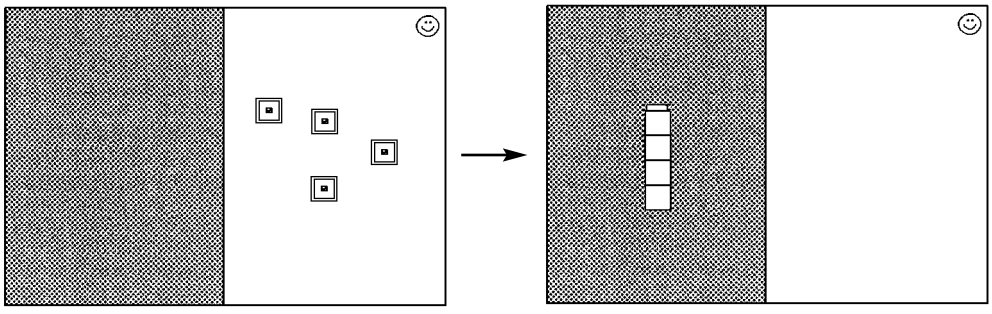
Next, ask the children to say what they have on the board by telling how many zibs and how many ones there are. (You want the children to tell how many of each at each step so that later they will be able to write each step and look for the patterns that emerge.)

How many do you have on your board now?

Zero zibs and one.

Initially, say what is on the board, for example, "zero zibs and one," *with* the children until the children pick up on what they are supposed to say and are able to say it without you.

Keep on saying "Plus one" and asking "How many?" ("zero zibs and two, zero zibs and three…") until there are four cubes, or one zib, on the white side of the board. Then have the children snap the cubes together and put the newly formed zib on the green side of the board.

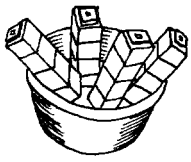


The first time you work with this activity, you will need to emphasize when it is time to make a group (a zib, in this example). After the children get the idea, they will be anticipating when they have enough to make a group and will be saying things like, "We have another zib!"

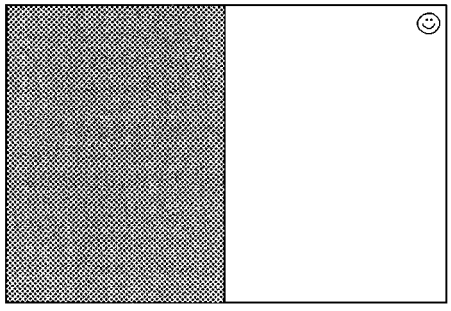
Continue the game, saying "Plus one" and asking "How many?" until the children each have three zibs and three on their board.

NOTE: In the beginning, it will be easier for the children if they do not have to regroup beyond two places. If you were to continue to say "Plus one" after the children have three zibs and three on their boards, the children would then have another zib. If they put that zib on the green side of the board, then they would have four zibs. Whenever the children reach four zibs, they must regroup the four zibs into another group, placing the zibs into a container to the left of the board to form a new group called a *big zib*.

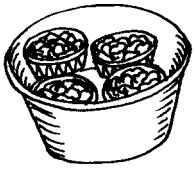
Using Cubes



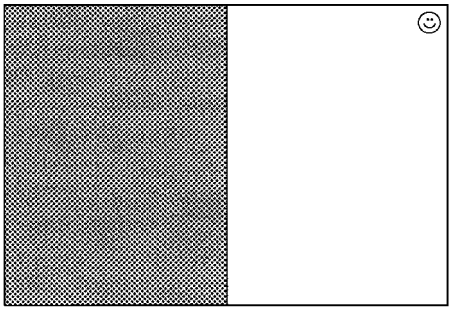
Big Zib



Using Beans



Big Zib

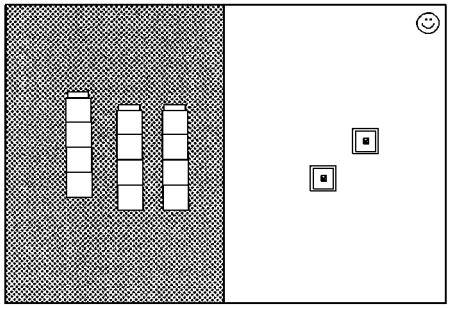


To avoid dealing with big zibs in these first lessons, stop saying "Plus one" when three zibs and three is reached. See activity 1–4 for a description of what to do when you are ready to extend the regrouping beyond two places.

The Minus-One Game

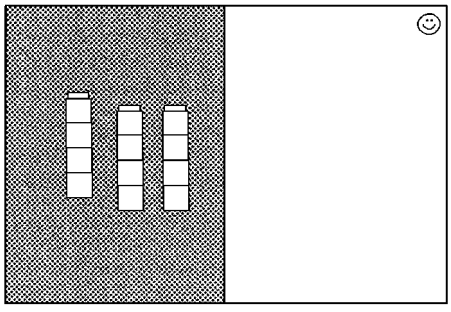
When three zibs and three is reached, tell the children that you are going to start saying "Minus one."

Minus one. How many do we have now?



Three zibs and two.

Continue saying "Minus one" until you reach three zibs and zero.



At this point, talk to the children about what they can do to take one cube away. Some will naturally break up one zib into four loose cubes from which they can take one away. Others will break off one cube and leave the rest of the zib on the green side of the board. Help the children see that three joined cubes no longer form a zib, so they can't remain on the green side. For example:

If you take one cube off a zib, is it still a zib?

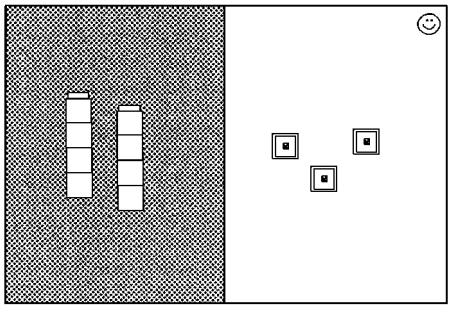
No. There's only three.

Only zibs belong on the green side—so what shall we do?

Put three cubes on the white side.

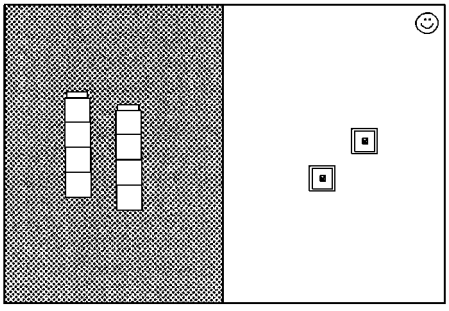
Yeah, but first we have to break them up.

How many zibs do we have now?



Two zibs and three.

Minus one. How many?



Two zibs and two.

Continue saying "Minus one" until you reach zero zibs and zero.

Repeat this activity on succeeding days. Vary it sometimes by using beans and cups with the place-value boards.

You do not want the children to think that the grouping games are played only with groups of four, so after two to five more lessons, begin playing with groups of other sizes as described in the following activity.