Understanding the Equal Sign: Part 1

Lesson Overview

Students explore *equivalence* and the development of a relational understanding of the equal sign. By exploring equations written in forms other than standard form, students come to understand the equal sign as indicating two quantities are equivalent. A relational understanding of the equal sign is critical to their future work with equations.

Vocabulary

equal sign

equation

expression

Materials

Pan balance or number balance

Lesson Objectives

- Develop a relational understanding of the equal sign by identifying equations written in various forms (other than $a + b = c$) as true orfalse and reasoning why.

Rationale for the Tasks

- Students are familiar with equations written in standard form (for example, $a + b = c$). It makes sense to begin with an equation of that form ($4 + 6 = 10$) when introducing the concept of true and false equations.
- The equation $4 + 6 = 10 + 0$ builds from the easily accepted $4 + 6 = 10$ to encourage students to think about operations on both sides of the equal sign.
- The equation $10 = 4 + 6$ asks students to think about an equation they may refer to as "backwards." More questioning is needed to see if the student has an operational or relational view of the equal sign.
- The equation $10 = 10$ challenges students who feel uncomfortable with equations where there is "nothing to do" (no operation). This reveals the misconception that equations are about performing calculations and that the equal sign means "the answer comes next."
- The remaining equations in the student activity encourage students to consider equations with operations on both sides. Students often find this to be the most challenging form, so consistent exposure is key to building students' relational understanding of the equal sign.
- Students are asked to write their own true/false equations because this activity can be motivating for them while revealing the extent to which they are comfortable working with equations in various forms.

Lesson at a Glance

Big Idea

Students learn to think relationally about the equal sign by identifying equations as true or false and reasoning why.

Jumpstart

How would you describe what this symbol means?

=

Explore and Discuss

Have students complete Tasks 1 and 2 and discuss in small groups what makes an equation true or false. Have students complete Task 3 and discuss in their groups what constitutes a good working definition of the equal sign. Do students see the equal sign as an operational symbol, or have they begun to develop a relational understanding of the equal sign?

Sit with groups and observe what they are saying. Select students who exhibit different ways of thinking to enrich whole-group discussions. Help groups that need support by using discussion-building strategies (see Teaching Support, page 7).

Review and Discuss

Are these equations true or false? Explain.

$11 + 4 = 15 + 2$

$23 = 17 + 6$

Jumpstart

Write or display the Jumpstart question on the board.

Listen to students' responses. Do students say that the equal sign means "the answer" or the "total"?
Do they say the equal sign means the "equation is balanced," or that "one side has the same value as the other side"?

Help students understand that the quantities on either side of the equal sign are the same amount or have the same value.

Jumpstart

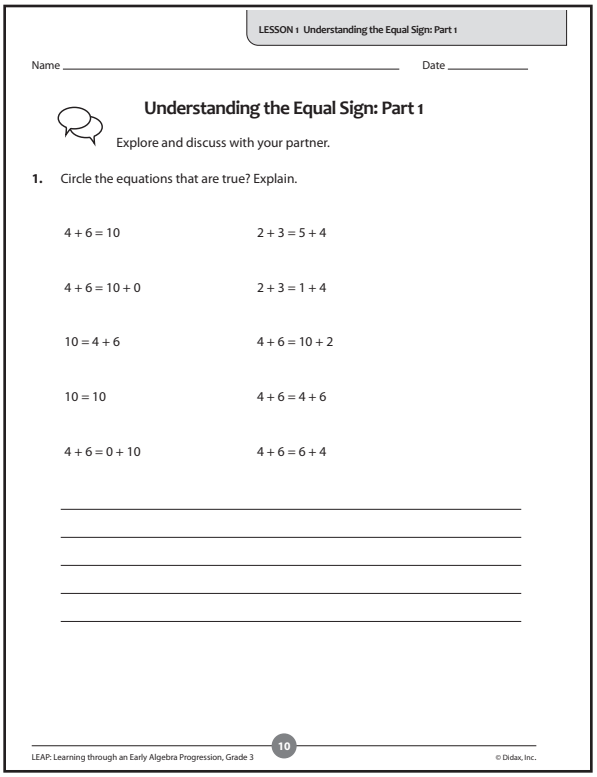
How would you describe what this symbol means?

=

Explore and Discuss

1. **What can you say about all the equations that are true?**
   Think about how students' responses relate to their definitions of the equal sign discussed in the Jumpstart. Use these responses to reinforce the notion that the symbol = indicates that two quantities have the same value.

   You may want to show the equal sign as the middle of a pan balance, where the objects on either side have the same weight. You might also show it as the middle of a number balance, where the expressions indicated on either side have the same value. Tools such as a pan balance or number balance can help students explore equivalence in concrete ways and can increase understanding. Gesturing to convey balance can also support students in picturing what it means for two quantities to be equivalent.



LESSON 1 Understanding the Equal Sign: Part 1

Name _____ Date _____

Understanding the Equal Sign: Part 1

Explore and discuss with your partner.

1. Circle the equations that are true? Explain.

4 + 6 = 10	2 + 3 = 5 + 4
4 + 6 = 10 + 0	2 + 3 = 1 + 4
10 = 4 + 6	4 + 6 = 10 + 2
10 = 10	4 + 6 = 4 + 6
4 + 6 = 0 + 10	4 + 6 = 6 + 4

10

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2. How did you determine whether your partner's equations were true or false?

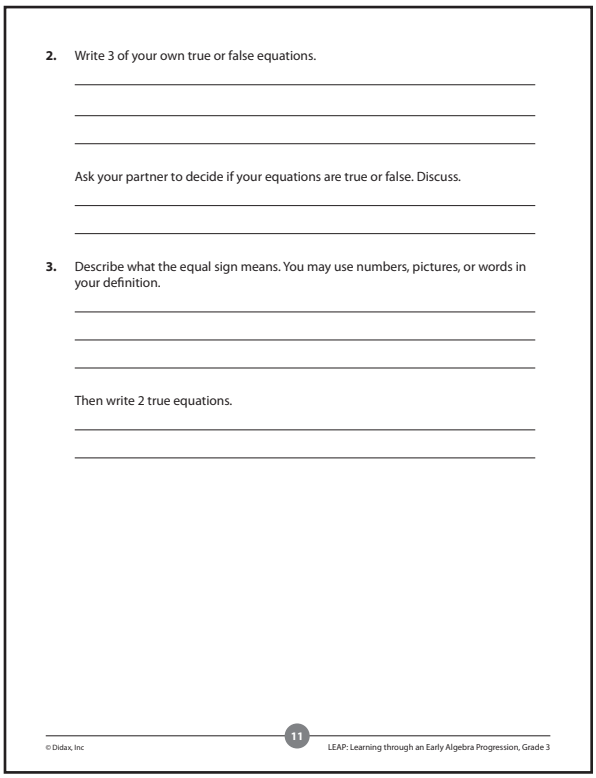
Have students trade their true/false equations with a partner and discuss. Share some of the equations with the whole class, including those that are in forms other than standard form (4 + 6 = 10). Make sure to include those with operations on both sides of the equal sign (such as 4 + 6 = 5 + 5), with operations on only one side of the equal sign (such as 8 = 3 + 5 or 10 + 2 = 12), and with no operation at all (such as 9 = 9).

3. How did you describe what the equal sign means?

At this point, you should hear students beginning to talk relationally about the equal sign. Do they say, for example, "The equal sign means that whatever amount you have on the left, you have that same amount on the right" or "The equal sign means the sides are balanced"?

Listen for students who still hold an operational view of the equal sign—that is, they think the equal sign means they need to find the answer.

Notice whether students are comfortable writing equations in different forms other than standard form.



2. Write 3 of your own true or false equations.

Ask your partner to decide if your equations are true or false. Discuss.

3. Describe what the equal sign means. You may use numbers, pictures, or words in your definition.

Then write 2 true equations.

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Thinking about Student Responses

Students' understanding of the equal sign as an operational symbol ("the answer comes next") often leads them to be unsure about any equation that is not in a standard form such as $a + b = c$.

For example, they may say that 8 = 8 is false because there is "nothing to do," or that 9 = 5 + 4 is "backwards" because the operation is to the right of the equal sign. They may also ignore the last number in the expression on the right side of the equal sign, saying, for example that 2 + 3 = 1 + 4 is false because 2 + 3 ≠ 1 and 4 + 6 = 10 + 2 is true because 4 + 6 = 10. They might also say 4 + 6 = 10 + 2 can be rewritten as 4 + 6 = 10 + 2 = 12.

In contrast, students who hold a relational view will likely compute each side of the equation separately and then compare the quantities. For example, they would reason that 2 + 3 = 1 + 4 is true because 2 + 3 and 1 + 4 are each equal to 5.

Even very young students can develop a relational understanding of the equal sign when given the opportunity to reason about true/false equations in various forms.

Review and Discuss

Are the following equations true or false? Explain.

$11 + 4 = 15 + 2$

$23 = 17 + 6$

Review and Discuss

Write or display the Review and Discuss question on the board.

- Listen for students who still think operationally about the equal sign. Such students might say, "11 + 4 = 15 + 2 is true because 11 + 4 = 15." You could ask, "What about the 2? What role does it play in the equation?"
- Do students think that equations cannot have a single value to the left of the equal sign? Listen for students who say, "23 = 17 + 6 is backwards." Help students understand that 23 = 17 + 6 is equivalent to 17 + 6 = 23.
- Students are beginning to think relationally about the equal sign when they understand that an equation is true only if the values of the quantities or expressions on both sides are the same.

Addressing Common Difficulties

Many students with an operational understanding of the equal sign believe that the equal sign means, "Gives the total" or "The answer comes next." Students might say, "2 + 3 = 5 = 1 is a true equation" because they add the expression on left side (the sum of which is 5) and ignore the + 1 in the expression on the right.

To counter these difficulties, ensure that students understand the equal sign means balance. You could say, "The equal sign means that whatever amount you have on the left is the same as what you have on the right," or "2 + 3 = 5 + 1 is not a true equation because 2 + 3 is 5, but 5 + 1 is 6 so this equation isn't balanced." For students who need more support, use tools such as a number balance to explore equivalence in more concrete ways.

Teaching Support

Developing a Relational Understanding of the Equal Sign

You might assume that the meaning of the equal sign is fairly straightforward and that nothing more than a simple explanation is needed. Research has shown, however, that an operational view of the equal sign is an entrenched notion for many students. Students require exposure to equations in various forms and discussions over time to develop a correct, relational understanding of the equal sign. The ideas presented here will be revisited in the Grade 3 lessons to come.

SSL Supporting Struggling Learners

For more information, refer to Figure 1: Strategies to Support Struggling Learners.

Challenging "Equation Strings"

Challenge students to think about the validity of the "equation strings" they often write as they record their mathematical calculations. For example, a student who is asked to add 12 + 27 + 15 might represent their action of first adding 12 and 27, and then adding 15 to that result, as the "equation string" 12 + 27 = 39 + 15 = 54. Ask students whether such equation strings make sense and are mathematically correct statements.

Focusing on Relationships between Quantities

Encourage students to test whether an equation is true or false by thinking about the quantities on either side of the equal sign. For example, in the equation 3 + 10 = 4 + 9, a student might reason, "Since 4 is one more than 3 and 9 is one less than 10, then the equation is true." Such a strategy, known as compensation, relies on an important understanding of how quantities are related rather than on computational work.

Mathematical Convention

The equal sign is a mathematical symbol used to represent the equivalence of two quantities or mathematical expressions.

Name _____ Date _____

Understanding the Equal Sign: Part 1

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$4 + 6 = 10 + 0$	$2 + 3 = 1 + 4$
$10 = 4 + 6$	$4 + 6 = 10 + 2$
$10 = 10$	$4 + 6 = 4 + 6$
$4 + 6 = 0 + 10$	$4 + 6 = 6 + 4$

2. Write 3 of your own true or false equations.

Ask your partner to decide if your equations are true or false. Discuss.

3. Describe what the equal sign means. You may use numbers, pictures, or words in your definition.

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